10/17/19

M157 Exercises: (See next page)

#4) (a) See notes from 10/15/19.

(b) $e_{50:\overline{10|}} = \sum_{k=1}^{10} {}_kP_{50}$

DMh($\omega = 120$) $\Rightarrow$ ${}_kP_{50} = \frac{70-k}{70}$

$e_{50:\overline{10|}} = \frac{69}{70} + \frac{68}{70} + \frac{67}{70} + \cdots + \frac{60}{70}$

$= \frac{69/70 + 60/70}{2} \cdot 10 = \frac{129}{14}$

value of an arithmetic sum

$= (\text{Avg. of } 1^{st} \text{ \& last term}) \cdot (\# \text{ of terms})$

#6) $e_{xy:\overline{20|}} = \sum_{k=1}^{20} {}_kP_{xy}$

$= (1.02)^{-1} + (1.02)^{-2} + \cdots + (1.02)^{-20}$

geometric w/ $r = 1.02^{-1}$

$v = \frac{1}{1.02}$ $\quad v + v^2 + \cdots + v^{20} = a_{\overline{20|}.02} = 16.35\ldots$

or $\frac{(1.02)^{-1} - (1.02)^{-21}}{1 - (1.02)^{-1}}$

value of a geometric sum

$= \frac{1^{st}\text{ term} - 1^{st}\text{ omitted term}}{1 - \text{ratio}}$

L-TAM Module 1 Section 7 Exercises

1. Given ${}_tp_x = e^{-.02t}$, determine

   (a) $\overset{o}{e}_x$

   (b) e_x

2. Given ${}_tp_{\overline{xy}} = e^{-.02t}$, determine

   (a) $\overset{o}{e}_{\overline{xy}}$

   (b) $e_{\overline{xy}}$

3. Determine the value of $T_{\overline{xy}}$ if $T_x + T_y = 40$ and $T_xT_y = 346.71$.

4. Given mortality for (50) follows a DML(120) model, determine

   (a) $\overset{o}{e}_{50:\overline{10|}}$

   (b) $e_{50:\overline{10|}}$

5. Given mortality for (x) follows a CF($\mu = .025$) model, determine

   (a) $\overset{o}{e}_{x:\overline{5|}}$

   (b) $e_{x:\overline{5|}}$

6. Given ${}_tp_{xy} = (1.02)^{-t}$, determine $e_{xy:\overline{20|}}$

7. Given $q_{80} = .05$ and $q_{81} = .10$

   (a) determine $e_{80:\overline{2|}}$

   (b) if $e_{80} = 6.08$, determine e_{82}

M1S8: Independent Lives

(Assume T_x & T_y are independent random variables)

Recall:

(Joint-Life Status) $\sigma = xy$ $\quad T_{xy} = \text{Min}(T_x, T_y)$

(Last-Survivor Status) $\sigma = \overline{xy}$ $\quad T_{\overline{xy}} = \text{Max}(T_x, T_y)$

Formulas:

1) $Pr(T_{xy} > n) = Pr(T_x > n \text{ \& } T_y > n) \overset{\text{ind.}}{=} Pr(T_x > n) \cdot Pr(T_y > n)$

$\therefore \quad {}_nP_{xy} = {}_nP_x \cdot {}_nP_y$

Then $\quad {}_nq_{xy} = 1 - {}_nP_{xy}$

2) $Pr(T_{\overline{xy}} \leq n) = Pr(T_x \leq n \text{ \& } T_y \leq n) = Pr(T_x \leq n) \cdot Pr(T_y \leq n)$

$\therefore \quad {}_nq_{\overline{xy}} = {}_nq_x \cdot {}_nq_y$

Then $\quad {}_nP_{\overline{xy}} = 1 - {}_nq_{\overline{xy}}$

Remarks: 1) ${}_nP_{\overline{xy}} = 1 - {}_nq_x \cdot {}_nq_y = 1 - (1 - {}_nP_x) \cdot (1 - {}_nP_y)$

$= 1 - (1 - {}_nP_x - {}_nP_y + {}_nP_x \cdot {}_nP_y)$

$= {}_nP_x + {}_nP_y - {}_nP_{xy}$ (Same as $(\,)_x + (\,)_y = (\,)_{xy} + (\,)_{\overline{xy}}$)

2) $$\mathring{e}_{\overline{xy}} = \int_0^\infty {}_tp_{\overline{xy}}\,dt$$

$$= \int_0^\infty \left({}_tp_x + {}_tp_y - {}_tp_{xy}\right) dt$$

$$= \mathring{e}_x + \mathring{e}_y - \mathring{e}_{xy}$$

Recall Notation for Force of Mortality:

$$\mu_{x+t} = \mu_x(t)$$

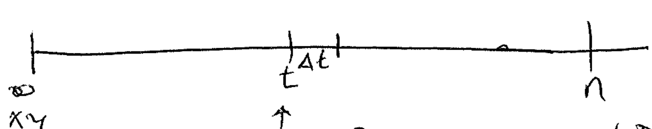
$$\mu_{y+t} = \mu_y(t)$$

Contingent Statuses

(For these statuses, we only study q's ; there are no p's.)

1) $$\underbrace{\sigma = \overset{1}{x}y}_{\text{status fails when (x) dies, if that happens before (y) dies}} \qquad \text{or} \qquad \underbrace{\tau = x\overset{1}{y}}_{\text{status fails when (y) dies, if that happens before (x) dies.}}$$

Consider: ${}_nq_{\overset{1}{x}y} \equiv Pr(\text{(x) dies first and within } n \text{ years})$



$Pr = {}_tp_{xy} \cdot \mu_x(t) \cdot \Delta t$ (Integrand)

$$\therefore \; {}_nq_{\overset{1}{x}y} = \int_0^n {}_tp_{xy} \cdot \mu_x(t)\,dt$$

2) $T = \overset{2}{x}y$ or $T = x\overset{2}{y}$

$$ {}_n q_{x\overset{2}{y}} = \int_0^n {}_t q_x \cdot {}_t p_y \cdot \mu_y(t) \cdot dt $$

Remarks:

1) ${}_n q_x = {}_n q_{\overset{1}{x}y} + {}_n q_{\overset{2}{x}y}$

2) ${}_n q_{xy} = {}_n q_{\overset{1}{x}y} + {}_n q_{x\overset{1}{y}}$

3) ${}_n q_{\overline{xy}} = {}_n q_{\overset{2}{x}y} + {}_n q_{x\overset{2}{y}}$

4) ${}_n q_{\overset{1}{x}y} - {}_n q_{x\overset{2}{y}} = {}_n q_x \cdot {}_n p_y$